

# Lambek Grammars as Second-order Abstract Categorical Grammars

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# Outline

## ► Motivation

Lambek Grammars and Algebras

Hypothetical Reasoning

Conclusions

Motivation

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# Main Question

Can second-order ACG faithfully represent Lambek Grammars?

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- ▶ *NO*, obviously

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- ▶ Hmm. . .

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# Lambek Grammar (LG)

A deductive (Post) system

Primitive types	$P$	$::= s, n, np$
Syntactic Types	$A, B$	$::= P \mid A \backslash B \mid B / A$
Environments	$\Gamma, \Delta$	$::= \bullet \mid A \mid A, \Gamma \mid \Gamma, A$
Judgements	$\Gamma \vdash A$	

A non-traditional variation of a less-common natural deduction presentation of Lambek Calculus and Grammar (LG) – to be called LA

LA is equivalent to the traditional LG



# Lambek Grammar/LA: Inference Rules

$$\frac{\Delta \vdash B/A \quad \Gamma \vdash A}{\Delta, \Gamma \vdash B} /e$$

$$\frac{\Gamma, A \vdash B}{\Gamma \vdash B/A} /i$$

$$\frac{\Gamma \vdash A \quad \Delta \vdash A \setminus B}{\Gamma, \Delta \vdash B} \setminus e$$

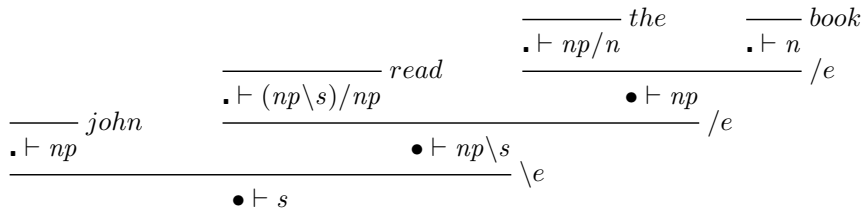
$$\frac{A, \Gamma \vdash B}{\Gamma \vdash A \setminus B} \setminus i$$

$$\frac{}{A \vdash A} \text{Var}$$

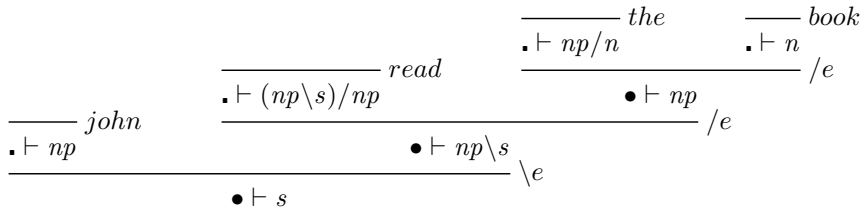
## LA: Lexical Items

$$\frac{}{\bullet \vdash np} \textit{john}$$
$$\frac{}{\bullet \vdash np/n} \textit{the}$$
$$\frac{}{\bullet \vdash n} \textit{book}$$
$$\frac{}{\bullet \vdash (np \setminus s)/np} \textit{read}$$

# Sample LA Derivation

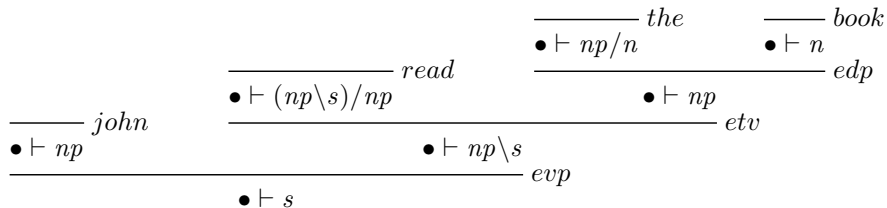


# Sample LA Derivation



Deduction as Grammar

# Sample LA Derivation'



Deduction as Grammar

# Algebra

evp :  $\langle \bullet; np \rangle \rightarrow \langle \bullet; vp \rangle \rightarrow \langle \bullet; s \rangle$   
edp :  $\langle \bullet; det \rangle \rightarrow \langle \bullet; n \rangle \rightarrow \langle \bullet; np \rangle$   
etv :  $\langle \bullet; tv \rangle \rightarrow \langle \bullet; np \rangle \rightarrow \langle \bullet; vp \rangle$

john :  $\langle \bullet; np \rangle$   
book :  $\langle \bullet; n \rangle$   
the :  $\langle \bullet; det \rangle$   
read :  $\langle \bullet; tv \rangle$

evp john (etv read (edp the book))

## CFG in CNF

$\langle \bullet; s \rangle \rightarrow \langle \bullet; np \rangle \langle \bullet; vp \rangle$

$\langle \bullet; np \rangle \rightarrow \langle \bullet; det \rangle \langle \bullet; n \rangle$

$\langle \bullet; vp \rangle \rightarrow \langle \bullet; tv \rangle \langle \bullet; np \rangle$

$\langle \bullet; np \rangle \rightarrow \text{"john"}$

$\langle \bullet; n \rangle \rightarrow \text{"book"}$

$\langle \bullet; det \rangle \rightarrow \text{"the"}$

$\langle \bullet; tv \rangle \rightarrow \text{"read"}$

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Can second-order ACG faithfully represent Lambek Grammars?

- ▶ No, obviously
- ▶ Yes, obviously



# Main Question

Can second-order ACG faithfully represent Lambek Grammars?

- ▶ No, obviously
- ▶ Yes, obviously
- ▶ But what about the full LG?

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# Lambek Grammar/LA: Inference Rules

$$\frac{\Delta \vdash B/A \quad \Gamma \vdash A}{\Delta, \Gamma \vdash B} /e$$

$$\frac{\Gamma, A \vdash B}{\Gamma \vdash B/A} /i$$

$$\frac{\Gamma \vdash A \quad \Delta \vdash A \setminus B}{\Gamma, \Delta \vdash B} \setminus e$$

$$\frac{A, \Gamma \vdash B}{\Gamma \vdash A \setminus B} \setminus i$$

$$\frac{}{A \vdash A} Var$$

## Contra ACG

“The best approximations that we can obtain all suffer from overgeneration because non-commutativity is insufficiently enforced.” (Moot, 2014)

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- ▶ No (Kubota, Levin, Moot)

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Can second-order ACG faithfully represent Lambek Grammars?

- ▶ No, obviously
- ▶ Yes, obviously
- ▶ No (Kubota, Levin, Moot)
- ▶ Yes? (Pentus result)

# Main Question

Can second-order ACG faithfully represent Lambek Grammars?

- ▶ No, obviously
- ▶ Yes, obviously
- ▶ No (Kubota, Levin, Moot)
- ▶ No (Pentus shown only weak equivalence)



# Main Question

Can second-order ACG faithfully represent Lambek Grammars?

- ▶ No, obviously
- ▶ Yes, obviously
- ▶ No (Kubota, Levin, Moot)
- ▶ No (Pentus shown only weak equivalence)
- ▶ Yes (De Groote, 2016)

# De Groote, 2016 (Final)

MAN	: $n$
WOMAN	: $n$
SOME	: $n \rightarrow np$
SOME <sub>0</sub>	: $(np_0 \rightarrow n_0) \rightarrow np_1 \rightarrow np_2$
EVERY	: $n \rightarrow np$
EVERY <sub>0</sub>	: $(np_0 \rightarrow n_0) \rightarrow np_1 \rightarrow np_2$
LOVES	: $np \rightarrow np \rightarrow s$
LOVES <sub>0</sub>	: $np \rightarrow np_3 \rightarrow s_0$
LOVES <sub>1</sub>	: $np \rightarrow np_4 \rightarrow s_1$
LOVES <sub>2</sub>	: $(np_1 \rightarrow np_2) \rightarrow np \rightarrow np_4 \rightarrow s_1$
LOVES <sub>3</sub>	: $(np_1 \rightarrow np_2) \rightarrow np_5 \rightarrow np_6 \rightarrow s_2$
LOVES <sub>4</sub>	: $np_5 \rightarrow np_6 \rightarrow s_2$
WHO	: $(np_3 \rightarrow s_0) \rightarrow n \rightarrow n$
WHO <sub>0</sub>	: $(np_5 \rightarrow np_6 \rightarrow s_2) \rightarrow n \rightarrow np_0 \rightarrow n_0$

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- ▶ No, obviously
- ▶ Yes, obviously
- ▶ No (Kubota, Levin, Moot)
- ▶ No (Pentus shown only weak equivalence)
- ▶ Yes (De Groote, 2016)
- ▶ No (lexicon explosion, third order)

# Algebra

john	: $\langle \bullet; np \rangle$	evp	: $\langle \bullet; np \rangle \rightarrow \langle \bullet; vp \rangle \rightarrow \langle \bullet; s \rangle$
book	: $\langle \bullet; n \rangle$	enn	: $\langle \bullet; n \rangle \rightarrow \langle \bullet; pp \rangle \rightarrow \langle \bullet; n \rangle$
the	: $\langle \bullet; det \rangle$	edp	: $\langle \bullet; det \rangle \rightarrow \langle \bullet; n \rangle \rightarrow \langle \bullet; np \rangle$
that	: $\langle \bullet; rel \rangle$	etv	: $\langle \bullet; tv \rangle \rightarrow \langle \bullet; np \rangle \rightarrow \langle \bullet; vp \rangle$
read	: $\langle \bullet; tv \rangle$	ehtv	: $\langle \bullet; tv \rangle \rightarrow \langle \bullet, np; np \rangle \rightarrow \langle \bullet, np; vp \rangle$
vanished	: $\langle \bullet; vp \rangle$	hnp	: $\langle \bullet, np; np \rangle$
		ehvp	: $\langle \bullet; np \rangle \rightarrow \langle \bullet, np; vp \rangle \rightarrow \langle \bullet, np; s \rangle$
		irnp	: $\langle \bullet, np; s \rangle \rightarrow \langle \bullet; s/np \rangle$
		erel	: $\langle \bullet; rel \rangle \rightarrow \langle \bullet; s/np \rangle \rightarrow \langle \bullet; pp \rangle$

## De Groot, 2016 (LDER)

prod<sub>0</sub> :  $\langle det \rangle \rightarrow \langle n \rangle \rightarrow \langle np \rangle$

prod<sub>1</sub> :  $\langle tv \rangle \rightarrow \langle np \rangle \rightarrow \langle np \rangle \rightarrow \langle s \rangle$

prod<sub>2</sub> :  $\langle pp/vp \rangle \rightarrow \langle vp \rangle \rightarrow \langle n \rangle \rightarrow \langle n \rangle$

prod<sub>4</sub> :  $\langle tv \rangle \rightarrow \langle np \rangle \rightarrow \langle vp \rangle$

prod<sub>7</sub> :  $\langle det \rangle \rightarrow \langle n/np \rangle \rightarrow \langle np/np \rangle$

prod<sub>8</sub> :  $\langle pp/vp \rangle \rightarrow \langle tv \rangle \rightarrow \langle n \rangle \rightarrow \langle n/np \rangle$

prod<sub>9</sub> :  $\langle tv \rangle \rightarrow \langle np/np \rangle \rightarrow \langle tv \rangle$

man :  $\langle n \rangle$

For comparison, us:

edp :  $\langle \bullet; det \rangle \rightarrow \langle \bullet; n \rangle \rightarrow \langle \bullet; np \rangle$

evp :  $\langle \bullet; np \rangle \rightarrow \langle \bullet; vp \rangle \rightarrow \langle \bullet; s \rangle$

etv :  $\langle \bullet; tv \rangle \rightarrow \langle \bullet; np \rangle \rightarrow \langle \bullet; vp \rangle$

enn :  $\langle \bullet; n \rangle \rightarrow \langle \bullet; pp \rangle \rightarrow \langle \bullet; n \rangle$

esrel :  $\langle \bullet; pp/vp \rangle \rightarrow \langle \bullet; vp \rangle \rightarrow \langle \bullet; pp \rangle$

man :  $\langle \bullet; n \rangle$

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- ▶ No, obviously
- ▶ Yes, obviously
- ▶ No (Kubota, Levin, Moot)
- ▶ No (Pentus shown only weak equivalence)
- ▶ Yes (De Groote, 2016)
- ▶ No (lexicon explosion, third order)
- ▶ Yes! (finite hyp-rank)

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## Conclusion

For any LG and the natural number  $n$ , there exists a CFG whose derivations are all and only LG derivations of hyp-rank  $n$ . The LG lexicon enters CFG as is, with no duplications, let alone exponential explosions.

LG of a bounded hyp-rank are *strongly* equivalent to CFG