Lambek Grammars as Second-order Abstract Categorial Grammars

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LENLS
November 12, 2019

Outline

Motivation

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Main Question

Can second-order ACG faithfully represent Lambek Grammars?

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- *NO*, obviously

Main Question

Can second-order ACG faithfully represent Lambek Grammars?

- *NO*, obviously
- Hmm. . .

Outline

Lambek Grammar (LG)

A deductive (Post) system

Primitive types	P	$::=$	s, n, np
Syntactic Types	A, B	$::=$	$P \mid A\backslash B \mid B/A$
Environments	Γ, Δ	$::=$	$\centerdot \mid A \mid A, \Gamma \mid \Gamma, A$
Judgements	$\Gamma \vdash A$		

A non-traditional variation of a less-common natural deduction presentation of Lambek Calculus and Grammar (LG) – to be called LA

LA is equivalent to the traditional LG

Lambek Grammar/LA: Inference Rules

$$\frac{\Delta \vdash B/A \quad \Gamma \vdash A}{\Delta, \Gamma \vdash B}\,/e \qquad \frac{\Gamma, A \vdash B}{\Gamma \vdash B/A}\,/i$$

$$\frac{\Gamma \vdash A \quad \Delta \vdash A\backslash B}{\Gamma, \Delta \vdash B}\,\backslash e \qquad \frac{A, \Gamma \vdash B}{\Gamma \vdash A\backslash B}\,\backslash i$$

$$\frac{}{A \vdash A}\,Var$$

LA: Lexical Items

$$\frac{}{\centerdot \vdash np}\, john \qquad \frac{}{\centerdot \vdash np/n}\, the \qquad \frac{}{\centerdot \vdash n}\, book \qquad \frac{}{\centerdot \vdash (np\backslash s)/np}\, read$$

Sample LA Derivation

$$\dfrac{\dfrac{}{. \vdash np}\,john \qquad \dfrac{\dfrac{}{. \vdash (np\backslash s)/np}\,read \qquad \dfrac{\dfrac{}{. \vdash np/n}\,the \qquad \dfrac{}{. \vdash n}\,book}{\bullet \vdash np}\,/e}{\bullet \vdash np\backslash s}\,/e}{\bullet \vdash s}\,\backslash e$$

Sample LA Derivation

$$
\dfrac{\dfrac{}{.\vdash np}\,john \qquad \dfrac{\dfrac{}{.\vdash (np\backslash s)/np}\,read \qquad \dfrac{\dfrac{}{.\vdash np/n}\,the \qquad \dfrac{}{.\vdash n}\,book}{\bullet\vdash np}\,/e}{\bullet\vdash np\backslash s}\,/e}{\bullet\vdash s}\,\backslash e
$$

Sample LA Derivation’

$$\dfrac{\dfrac{}{\bullet \vdash np}\,\mathit{john} \qquad \dfrac{\dfrac{}{\bullet \vdash (np\backslash s)/np}\,\mathit{read} \qquad \dfrac{\dfrac{}{\bullet \vdash np/n}\,\mathit{the} \qquad \dfrac{}{\bullet \vdash n}\,\mathit{book}}{\bullet \vdash np}\,\mathit{edp}}{\bullet \vdash np\backslash s}\,\mathit{etv}}{\bullet \vdash s}\,\mathit{evp}$$

Algebra

$$\begin{aligned}
\text{evp} &: \langle \bullet; np \rangle \to \langle \bullet; vp \rangle \to \langle \bullet; s \rangle \\
\text{edp} &: \langle \bullet; det \rangle \to \langle \bullet; n \rangle \to \langle \bullet; np \rangle \\
\text{etv} &: \langle \bullet; tv \rangle \to \langle \bullet; np \rangle \to \langle \bullet; vp \rangle
\end{aligned}$$

$$\begin{aligned}
\text{john} &: \langle \bullet; np \rangle \\
\text{book} &: \langle \bullet; n \rangle \\
\text{the} &: \langle \bullet; det \rangle \\
\text{read} &: \langle \bullet; tv \rangle
\end{aligned}$$

evp john (etv read (edp the book))

CFG in CNF

$$\begin{aligned}
\langle \bullet; s \rangle &\rightarrow \langle \bullet; np \rangle \ \langle \bullet; vp \rangle \\
\langle \bullet; np \rangle &\rightarrow \langle \bullet; det \rangle \ \langle \bullet; n \rangle \\
\langle \bullet; vp \rangle &\rightarrow \langle \bullet; tv \rangle \ \langle \bullet; np \rangle
\end{aligned}$$

$$\begin{aligned}
\langle \bullet; np \rangle &\rightarrow \texttt{"john"} \\
\langle \bullet; n \rangle &\rightarrow \texttt{"book"} \\
\langle \bullet; det \rangle &\rightarrow \texttt{"the"} \\
\langle \bullet; tv \rangle &\rightarrow \texttt{"read"}
\end{aligned}$$

Main Question

Can second-order ACG faithfully represent Lambek Grammars?

- No, obviously
- Yes, obviously

Main Question

Can second-order ACG faithfully represent Lambek Grammars?

- No, obviously
- Yes, obviously
- But what about the full LG?

Outline

Hypothetical Reasoning

$$\dfrac{\dfrac{}{. \vdash tv}\ \textit{read} \qquad np \vdash np}{\bullet, np \vdash vp}\ /e$$

$$\dfrac{\dfrac{}{. \vdash np}\ \textit{john} \qquad \bullet, np \vdash vp}{\bullet, np \vdash s}\ \backslash e$$

$$\dfrac{\bullet, np \vdash s}{\bullet \vdash s/np}\ /i$$

$$\dfrac{\dfrac{}{. \vdash rel}\ \textit{that} \qquad \bullet \vdash s/np}{\bullet \vdash pp}\ /e$$

$$\dfrac{\dfrac{}{. \vdash n}\ \textit{book} \qquad \bullet \vdash pp}{\bullet \vdash n}\ \backslash e$$

$$\dfrac{\dfrac{}{. \vdash det}\ \textit{the} \qquad \bullet \vdash n}{\bullet \vdash np}\ /e$$

$$\dfrac{\bullet \vdash np \qquad \dfrac{}{. \vdash}}{\bullet \vdash s}$$

Lambek Grammar/LA: Inference Rules

$$\frac{\Delta \vdash B/A \quad \Gamma \vdash A}{\Delta, \Gamma \vdash B} /e \qquad \frac{\Gamma, A \vdash B}{\Gamma \vdash B/A} /i$$

$$\frac{\Gamma \vdash A \quad \Delta \vdash A\backslash B}{\Gamma, \Delta \vdash B} \backslash e \qquad \frac{A, \Gamma \vdash B}{\Gamma \vdash A\backslash B} \backslash i$$

$$\frac{}{A \vdash A} Var$$

Contra ACG

"The best approximations that we can obtain all suffer from overgeneration because non-commutativity is insufficiently enforced." (Moot, 2014)

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- No, obviously
- Yes, obviously
- No (Kubota, Levin, Moot)

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Can second-order ACG faithfully represent Lambek Grammars?

- No, obviously
- Yes, obviously
- No (Kubota, Levin, Moot)
- Yes? (Pentus result)

Main Question

Can second-order ACG faithfully represent Lambek Grammars?

- No, obviously
- Yes, obviously
- No (Kubota, Levin, Moot)
- No (Pentus shown only weak equivalence)

Main Question

Can second-order ACG faithfully represent Lambek Grammars?

- No, obviously
- Yes, obviously
- No (Kubota, Levin, Moot)
- No (Pentus shown only weak equivalence)
- Yes (De Groote, 2016)

De Groote, 2016 (Final)

$$
\begin{array}{ll}
\text{MAN} & : n \\
\text{WOMAN} & : n \\
\text{SOME} & : n \to np \\
\text{SOME}_0 & : (np_0 \to n_0) \to np_1 \to np_2 \\
\text{EVERY} & : n \to np \\
\text{EVERY}_0 & : (np_0 \to n_0) \to np_1 \to np_2 \\
\text{LOVES} & : np \to np \to s \\
\text{LOVES}_0 & : np \to np_3 \to s_0 \\
\text{LOVES}_1 & : np \to np_4 \to s_1 \\
\text{LOVES}_2 & : (np_1 \to np_2) \to np \to np_4 \to s_1 \\
\text{LOVES}_3 & : (np_1 \to np_2) \to np_5 \to np_6 \to s_2 \\
\text{LOVES}_4 & : np_5 \to np_6 \to s_2 \\
\text{WHO} & : (np_3 \to s_0) \to n \to n \\
\text{WHO}_0 & : (np_5 \to np_6 \to s_2) \to n \to np_0 \to n_0
\end{array}
$$

Main Question

Can second-order ACG faithfully represent Lambek Grammars?

- No, obviously
- Yes, obviously
- No (Kubota, Levin, Moot)
- No (Pentus shown only weak equivalence)
- Yes (De Groote, 2016)
- No (lexicon explosion, third order)

Algebra

$$
\begin{aligned}
\text{john} &: \langle \bullet; np \rangle \\
\text{book} &: \langle \bullet; n \rangle \\
\text{the} &: \langle \bullet; det \rangle \\
\text{that} &: \langle \bullet; rel \rangle \\
\text{read} &: \langle \bullet; tv \rangle \\
\text{vanished} &: \langle \bullet; vp \rangle
\end{aligned}
$$

$$
\begin{aligned}
\text{evp} &: \langle \bullet; np \rangle \to \langle \bullet; vp \rangle \to \langle \bullet; s \rangle \\
\text{enn} &: \langle \bullet; n \rangle \to \langle \bullet; pp \rangle \to \langle \bullet; n \rangle \\
\text{edp} &: \langle \bullet; det \rangle \to \langle \bullet; n \rangle \to \langle \bullet; np \rangle \\
\text{etv} &: \langle \bullet; tv \rangle \to \langle \bullet; np \rangle \to \langle \bullet; vp \rangle \\
\text{ehtv} &: \langle \bullet; tv \rangle \to \langle \bullet, np; np \rangle \to \langle \bullet, np; vp \rangle \\
\text{hnp} &: \langle \bullet, np; np \rangle \\
\text{ehvp} &: \langle \bullet; np \rangle \to \langle \bullet, np; vp \rangle \to \langle \bullet, np; s \rangle \\
\text{irnp} &: \langle \bullet, np; s \rangle \to \langle \bullet; s/np \rangle \\
\text{erel} &: \langle \bullet; rel \rangle \to \langle \bullet; s/np \rangle \to \langle \bullet; pp \rangle
\end{aligned}
$$

De Groote, 2016 (LDER)

$$\begin{aligned}
\mathrm{prod}_0 &: \texttt{<}det\texttt{>} \to \texttt{<}n\texttt{>} \to \texttt{<}np\texttt{>} \\
\mathrm{prod}_1 &: \texttt{<}tv\texttt{>} \to \texttt{<}np\texttt{>} \to \texttt{<}np\texttt{>} \to \texttt{<}s\texttt{>} \\
\mathrm{prod}_2 &: \texttt{<}pp/vp\texttt{>} \to \texttt{<}vp\texttt{>} \to \texttt{<}n\texttt{>} \to \texttt{<}n\texttt{>} \\
\mathrm{prod}_4 &: \texttt{<}tv\texttt{>} \to \texttt{<}np\texttt{>} \to \texttt{<}vp\texttt{>} \\
\mathrm{prod}_7 &: \texttt{<}det\texttt{>} \to \texttt{<}n/np\texttt{>} \to \texttt{<}np/np\texttt{>} \\
\mathrm{prod}_8 &: \texttt{<}pp/vp\texttt{>} \to \texttt{<}tv\texttt{>} \to \texttt{<}n\texttt{>} \to \texttt{<}n/np\texttt{>} \\
\mathrm{prod}_9 &: \texttt{<}tv\texttt{>} \to \texttt{<}np/np\texttt{>} \to \texttt{<}tv\texttt{>} \\
\mathrm{man} &: \texttt{<}n\texttt{>}
\end{aligned}$$

For comparison, us:

$$\begin{aligned}
\mathrm{edp} &: \langle \bullet; det \rangle \to \langle \bullet; n \rangle \to \langle \bullet; np \rangle \\
\mathrm{evp} &: \langle \bullet; np \rangle \to \langle \bullet; vp \rangle \to \langle \bullet; s \rangle \\
\mathrm{etv} &: \langle \bullet; tv \rangle \to \langle \bullet; np \rangle \to \langle \bullet; vp \rangle \\
\mathrm{enn} &: \langle \bullet; n \rangle \to \langle \bullet; pp \rangle \to \langle \bullet; n \rangle \\
\mathrm{esrel} &: \langle \bullet; pp/vp \rangle \to \langle \bullet; vp \rangle \to \langle \bullet; pp \rangle \\
\mathrm{man} &: \langle \bullet; n \rangle
\end{aligned}$$

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Can second-order ACG faithfully represent Lambek Grammars?

- No, obviously
- Yes, obviously
- No (Kubota, Levin, Moot)
- No (Pentus shown only weak equivalence)
- Yes (De Groote, 2016)
- No (lexicon explosion, third order)
- Yes! (finite hyp-rank)

Outline

Conclusion

For any LG and the natural number n, there exists a CFG whose derivations are all and only LG derivations of hyp-rank n.

The LG lexicon enters CFG as is, with no duplications, let alone exponential explosions.

LG of a bounded hyp-rank are *strongly* equivalent to CFG