Compositional semantics of
same, different, total

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We propose a strictly compositional and uniform treatment for the internal readings of the symmetric predicates such as *same* and *different* and for the summative phrases such as *a total of*. The analyses handle multiple occurrences of symmetric and summative predicates in the same sentence, alongside of multiple quantificational elements. We treat symmetric predicates and summative phrases as wide-scope generalized existential quantifiers for choice functions. Like *Barker-parasitic* we treat symmetric and summative expressions as generalized quantifiers and relate them to universal quantification. However, our quantifiers scope wide rather than parasitically and avoid Barker’s non-standard interpretation of universal quantification. We introduce the analyses in terms of the familiar Quantifier Raising framework, which we make more precise and treat as a somewhat informal version of a type-logical grammar.
Puzzles

- (1) John and Bill read the same book.
What are the puzzles. Here is a typical sentence. It is ambiguous: suppose it occurs in a paragraph that starts: “Kim is reading Harry Potter....”. Then (1) will probably mean that John and Bill have read that magic novel. This is an external, or deictic reading. But (1) can have a meaning all by itself: there is an unnamed book that John has read, and Bill also has read. In other words, the set of books read by Bill and John has one book in common. This is the internal reading, the topic of our main interest.
Puzzles

- (1) John and Bill read the same book.
- (1a) The same waiter served everyone.

- (2) John and Bill read different books.
- (3) John and Bill went to the same school in the same city, had different majors but were members of the same club. . .
- (4) John read and Bill reviewed different books. I gave the same book to John on Wednesday and to Bill on Friday.
- (5) John gambled and Bill lost the total of 10,000.
- (5a) John gambled and Bill lost the total of 10,000 in the same casino but at different games. . .
Here is another example, with a quantifier. It is just as ambiguous. On internal reading, there is some unnamed waiter that has served all guests.
Puzzles

- (1) John and Bill read the same book.
- (1a) The same waiter served everyone.
- (1b) John and Bill read similar books.
Sentences of that type are very common; we can have “same”, “nearly identical”, “roughly the same”, etc., etc.
Puzzles

- (1) John and Bill read the same book.
- (1a) The same waiter served everyone.
- (1b) John and Bill read similar books.
- (2) John and Bill read different books.
And then we have something similar, but different.
Puzzles

- (1) John and Bill read the same book.
- (1a) The same waiter served everyone.
- (1b) John and Bill read similar books.
- (2) John and Bill read different books.
- (3) John and Bill went to the same school in the same city, had different majors but were members of the same club...
And we can stack them up, and up and up...
(1) John and Bill read the same book.
(1a) The same waiter served everyone.
(1b) John and Bill read similar books.
(2) John and Bill read different books.
(3) John and Bill went to the same school in the same city, had different majors but were members of the same club.
(4) John read and Bill reviewed different books.
I gave the same book to John on Wednesday and to Bill on Friday.
Same and different can appear with variety of complications: we have seen quantification, and here we see RNR and gapping.
Puzzles

- (1) John and Bill read the same book.
- (1a) The same waiter served everyone.
- (1b) John and Bill read similar books.
- (2) John and Bill read different books.
- (3) John and Bill went to the same school in the same city, had different majors but were members of the same club...
- (4) John read and Bill reviewed different books. I gave the same book to John on Wednesday and to Bill on Friday.
- (5) John gambled and Bill lost the total of 10,000.
The final piece of the puzzle is summatives such as total. Here 10,000 is the sum of John wins and Bill losses.
Puzzles

- (1) John and Bill read the same book.
- (1a) The same waiter served everyone.
- (1b) John and Bill read similar books.
- (2) John and Bill read different books.
- (3) John and Bill went to the same school in the same city, had different majors but were members of the same club.
- (4) John read and Bill reviewed different books. I gave the same book to John on Wednesday and to Bill on Friday.
- (5) John gambled and Bill lost the total of 10,000.
- (5a) John gambled and Bill lost the total of 10,000 in the same casino but at different games.
And again, summatives and same/different can stack up and up and up...
Puzzles

- (1) John and Bill read the same book.
- (1a) The same waiter served everyone.
- (1b) John and Bill read similar books.
- (2) John and Bill read different books.
- (3) John and Bill went to the same school in the same city, had different majors but were members of the same club.
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- (5a) John gambled and Bill lost the total of 10,000 in the same casino but at different games.

Same, different, total are indeed a very common feature. They are also quite resistant to compositional analyses. There was even a mathematical proof that one cannot have a compositional analysis with generalized quantifiers. The field has changed when this paper came out in 2007. Incidentally, in 2007 I attended my first ESSLII, in Dublin. Chris was teaching there. I recall one morning he came to breakfast saying that he just uploaded two new papers to Semantic Archive. One of them was Parasitic Scope. I guess that was the event that by mysterious cosmic connection forced me to take the same topic eight years later. The parasitic scope paper have dealt mostly with the single “same”, the example (1) and (1a). It offered a few thoughts on different, although without the detailed analysis.
Puzzles

➢ (1) John and Bill read the same book.
➢ (1a) The same waiter served everyone.
➢ (1b) John and Bill read similar books.
➢ (2) John and Bill read different books.
➢ (3) John and Bill went to the same school in the same city, had different majors but were members of the same club...
➢ (4) John read and Bill reviewed different books. I gave the same book to John on Wednesday and to Bill on Friday.
➢ (5) John gambled and Bill lost the total of 10,000.
➢ (5a) John gambled and Bill lost the total of 10,000 in the same casino but at different games...

This work: the unified treatment of all phenomena plus of internal and external readings
This work handles all these puzzles, plus the internal and external readings, *uniformly*. 
“There seem to be two separate points that the paper aims to establish, one relevant for the workshop theme and the other not so much, and as far as the novel contribution of this paper is concerned, the latter is by far the more important than the former. So, I am somewhat unsure whether I should recommend this paper to be included in the workshop program. The main point of the paper is a new analysis of symmetrical and summative predicates. This is largely independent of the question of what version of categorial grammar is optimal for the analysis of natural language . . .

If I got it right, the key proposal can be expressed in any version of categorial grammar equipped with a mechanism for handling quantification, such as. . . ” (Anonymous reviewer)
This paper is really looks like a black sheep in the workshop. I am truly grateful to the organizers for letting it in and attempting to fit it in. I still don’t quite understand the connection to dynamic semantics, maybe that will come later.

I must say that I do not view this work as the confrontation with parasitic scope: “Who is right?”. That is not the question. Taking inspiration from Chris work, perhaps all way back from 2007, I view the present paper as the further development and elaboration of his insights.

My intention for submitting to the workshop was to analyze same and different in my pet type logical grammar: NL with a slightly different semantic interpretation. And then I received reviews. A very insightful reviewer wrote the following.

However pains me to ditch my pet TLG, I took the reviewer advice and tried to explain the new analysis in a familiar framework, so that it could be easily understood and implemented in whatever categorial grammar people like. And we have seen many of them. For the framework, I chose the most familiar framework – perhaps too familiar. That is, I used the same framework that Chris Barker employed for most of his paper, also for reasons of its familiarity. This framework is Quantifier Raising (QR).
QR as an informal TLG

S

NP

John

VP

(NP\S)/NP

served

S/(NP\S)

everyone
I needed to make QR just a little bit more precise and less ad hoc, in order to express my analyses. One may view the end result as an informal TLG for semantic analyses. Let me take this simple example to illustrate it.
QR as an informal TLG

\[
\begin{array}{c}
S \\
S/(S^1) \\
\text{everyone} \\
\text{NP} \\
\text{John} \\
\text{VP/NP} \\
\text{served} \\
\text{NP} \\
\text{VP}^1 \\
\text{NP}^1 \\
\end{array}
\]
QR as an informal TLG

[1^{1}]  \quad e \rightarrow e \quad \lambda x : e. x

[VP^{1}]  \quad e \rightarrow (et) \quad \lambda x : e. \text{serve } x

[S^{1}]  \quad e \rightarrow t \quad \lambda x : e. \text{serve } x \text{ john}

[S/(S^{1})]  \quad (e \rightarrow t)t \quad \lambda C : (e \rightarrow t). \forall z. C \ z

[S]  \quad t \quad \forall z. \text{serve } z \text{ john}
I use an arrow to indicate hypothesis. You may read it as a sequent arrow.
Simple analysis of *same*

```
S
  S/S¹
/    
|     
everyone

S¹
  NP
  /    
 /     
Det The (S/(N\S))/N
| |    |    |    |    |    |    |
same waiter

VP¹
  /    
VP/NP NP¹
  /    | |    |
served 1¹
```
Let me start with an oversimplified analysis, to be improved later. I use a slightly different approach than that in the paper: now I move out “same waiter” whereas in the paper I moved only “same”. Which is better – you decide.
Simple analysis of *same*

The sentence structure is as follows:

- **S/S^2:N**
  - **same waiter**
  - **S/S^1**
    - **everyone**
    - **NP^2:N**
      - **Det**
        - **The**
      - **N^2:N**
        - **2^2:N**
    - **VP^1**
      - **served 1^1**
Simple analysis of *same*

\[
\begin{array}{c}
S \\
S/S^2:N \\
\text{same waiter} \\
\hline \\
S/S^1 \\
\text{everyone} \\
\hline \\
S^2:N \\
\end{array}
\]

\[
\begin{array}{c}
S^1;2:N \\
\hline \\
\text{NP}^2:N \\
\text{VP}^1 \\
\hline \\
\text{The } 2^{2:N} \\
\text{served } 1^{1}
\end{array}
\]

\[
\begin{align*}
[NP^2:N] &= (et) \to e \\
\text{[same waiter]} &= ((et) \to t)t \\
[S] &= t
\end{align*}
\]

\[
\begin{align*}
\lambda y : (et). \text{the } y \\
\lambda C : ((et) \to t). \exists i : e. \text{waiter } i \land \\
C \ (\lambda u : e. \text{waiter } u \land u = i) \\
\exists i : e. \text{waiter } i \land \\
\forall z. \text{serve } z \ (\text{the } (\lambda u. \text{waiter } u \land u = i))
\end{align*}
\]
I show the denotation for “same waiter”. It should be easy to figure out what “same” is. The equality stands for some equivalence relation.
Simple analysis of *same*

**Pro**

- *same* as a property of belonging to an equivalence class; a wide-scope existential quantifier for the reference entity of the class
- Multiple *same*

**Con**

- does not work for *different*
- *same* $\equiv$ *some*
Skolemization

\( \exists x. \forall y. P(x, y) \)

\( \forall y. \exists x. P(x, y) \)

\( \exists f. \forall y. P(f(y), y) \)

Disambiguation by imposing constraints on the chosen \( f \)
(e.g., requiring \( f \) to be a constant function)
Dependent quantification: Quantifying under hypotheses

Quantification operator (cf. *everyone*)
\[
qU : ((B \to t)t) = \lambda F : (B \to t). \forall z : B. F z
\]

Formula to quantify (cf. $S^{2:N,1:NP}$)
\[
p_{xy} = \lambda x : A. \lambda y : B. P(x, y)
\]

Simple quantification
\[
\lambda x : A. qU (\lambda y : B. p_{xy} x y)
\]

General case: dependent quantification
\[
\lambda f : (B \to A). qU (\lambda y : B. p_{xy} (f y) y)
\]
Let us now look back at how we used quantifiers in our derivations so far and attempt to abstract and generalize. The main question: how to quantify (and hence eliminate) one hypothesis when there may be others.

Quantification operator: like everyone. The formula to quantify is like $S^2:N,1:NP$ with two hypotheses. We want to quantify over $1:NP$. There is a simple way of doing it, which is what we have used. But there is a more general way.

The Skolem (choice) function $f$ expresses the possible correlation between the hypothesis. The simple quantification is obtained as a special case when $f$ is a constant function. We call this general procedure dependent quantification and use it extensively next.
General analysis of same

\[
[S^1:NP;2:N] \quad e \rightarrow (et) \rightarrow t \quad \lambda x: e. \lambda y: (et). \text{served } x \text{ (the } y) \\
[S^{2:NP}] \quad (e \rightarrow (et)) \rightarrow t \\
\lambda f: (e \rightarrow (et)). \forall z. \text{served } z \text{ (the } f z) \\
[\text{same waiter}] \quad ((e \rightarrow (et)) \rightarrow t)t \\
\lambda C. \exists i: e. \text{waiter } i \land C \quad (\lambda \gamma: e. \lambda u: e. \text{waiter } u \land u = i) \\
[S] \quad t \\
\exists i: e. \text{waiter } i \land \forall z. \text{serve } z \text{ (the } (\lambda u. \text{waiter } u \land u = i))
\]
Let us see how the dependent quantification works by re-doing our simple analysis.
See how types (parentheses) change when we move from $S^1:NP; 2:N$ to $S^2:N^{NP}$.

The end result, the meaning of the sentence, is the same as before. That should not be surprising. The choice function is the constant function: it receives as the argument the object $z$ chosen by the universal quantifier (‘the identity of the served person”) and disregards it.
General analysis of *same*

- **Category** \((S/(N\backslash S))/N\)
- **Trace named** \(n\) \(N^n\cdot N\)
- **Category of raised *same* \(N\)** \(S/S^n_{\cdot \cdot}^N\Gamma\)
- **Its semantic type** \(((\Gamma \rightarrow (et)) \rightarrow t)t\)
- **Denotation of *same* \(N\)
  \[\lambda C : ((\Gamma \rightarrow (et)) \rightarrow t). \exists i : e. \land P(i) \land C (\lambda \gamma : \Gamma. \lambda u : e. P(u) \land u = i)\]

Pro (as before)

- *same* as a property of belonging to an equivalence class; a wide-scope existential quantifier for the reference entity of the class
- Multiple *same*

- *same* \(\not\equiv\) *some*
  Raised *same* \(N\) is required to have dependent-quantification category
- now works for *different*
Let us recap, what has changed in the new analysis, and what remained the same. In the denotation, $P(x)$ is the property associated with $N$ in *same* $N$. 
General analysis of **different**

**same**

Category \( (S/(N\setminus S))/N \)

Trace named \( n \) \( N^n:N \)

Category of raised **same** \( N \) \( S/S^m:N^\Gamma \)

Its semantic type \( ((\Gamma \rightarrow (et)) \rightarrow t)t \)

Denotation of **same** \( N \)

\[ \lambda C : ((\Gamma \rightarrow (et)) \rightarrow t). \exists i : e. \land P(i) \land C \ (\lambda y : \Gamma. \lambda u : e. P(u) \land u = i) \]

**different**

Category \( (S/(N\setminus S))/N \)

Trace named \( n \) \( N^n:N \)

Category of raised **different** \( N \) \( S/S^m:N^\Gamma \)

Its semantic type \( ((\Gamma \rightarrow (et)) \rightarrow t)t \)

Denotation of **different** \( N \)

\[ \lambda C : ((\Gamma \rightarrow (et)) \rightarrow t). \exists p : (e \rightarrow (et)). (\forall u : \Gamma. \forall v : \Gamma. u \neq v \Rightarrow p(u) \cap p(v) = \emptyset) \land \]

\[ C (\lambda y : \Gamma. \lambda u : e. P(u) \land p(y)(u)) \]
Now, *same* can be extended to different and total: the choice function is not the constant function any more. Thus *different* finds such property $p$ indexed by $y : \Gamma$ that different indices correspond to non-intersecting properties.

Total is very similar, see the paper for details.
Multiple same/different

John and Bill bought the same book at different stores.
Since *same* and *different* have the same categories, they are freely interchangeable.
Discourse-wide quantification

**Internal same**

<table>
<thead>
<tr>
<th>Category</th>
<th>((S/(N\backslash S))/N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trace named (n)</td>
<td>(N^{n:N})</td>
</tr>
<tr>
<td>Category of raised same (N)</td>
<td>(S/S^{n:N\Gamma})</td>
</tr>
<tr>
<td>Its semantic type</td>
<td>(((\Gamma \rightarrow (et)) \rightarrow t)t)</td>
</tr>
<tr>
<td>Denotation of same (N)</td>
<td>(\lambda C : ((\Gamma \rightarrow (et)) \rightarrow t). \exists i : e. \land P(i) \land C (\lambda \gamma : \Gamma. \lambda u : e. P(u) \land u = i))</td>
</tr>
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</table>

**External same**

<table>
<thead>
<tr>
<th>Category</th>
<th>((S/(N\backslash S))/N)</th>
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<tbody>
<tr>
<td>Trace named (n)</td>
<td>(N^{n:N})</td>
</tr>
<tr>
<td>Category of raised same (N)</td>
<td>(S/S^{n:NNP'})</td>
</tr>
<tr>
<td>Its semantic type</td>
<td>(((e \rightarrow (et)) \rightarrow t)t)</td>
</tr>
<tr>
<td>Denotation of same (N)</td>
<td>(\lambda C : ((e \rightarrow (et)) \rightarrow t). C (\lambda \gamma : e. \exists i : e. \land i = \gamma \land \lambda u : e. P(u) \land u = i))</td>
</tr>
</tbody>
</table>

John read Emma. Bill read the same book.
Conclusions

- Uniform treatment of *same, different, total*: wide-scope quantifiers, over Skolem functions returning property
- Uniform treatment of internal and external readings
- Mechanism: dependent quantification
- Other motivations for choice functions

Relation to *Parasitic Scope*

- Same choice functions (of different types though)
- Same requirement of side universal quantification
- Our choice function is quantified wider (than the side universal)
- No need to postulate that the universal quantifier quantifies over groups of entities