

# Events and Relative Clauses

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**Abstract.** This work is the continuation of the development of polynomial event semantics (a dialect of Neo-Davidsonian event semantics), using the FraCaS textual entailment corpus as a whetstone. This time we grapple with various, often complicated, relative clauses.

Relative clauses have hardly been analyzed before in event semantics. Although simple cases are straightforward, challenges arise when a clause contains quantification, coordination or negation. We deal with such complications in the present paper, focusing on entailments.

## 1 Introduction

This work is the continuation of [5, 6, 7] on polynomial event semantics and textual entailment.

Deciding entailments ‘by pure logic’ – without resorting to meaning postulates – is one of the most attractive features of event semantics. However, beyond the classical “Brutus stabbed Caesar violently”, one quickly runs into problems. One is quantification, described and dealt with in [5, 7]; another is negation [6]. Then there are relative clauses, which are rarely considered in event semantics. In fact, the recent survey [10] and the extensive study [3] give, among the multitude of examples, not a single analysis of a sentence with a relative clause.

A relative clause appears already in the very first problem in the FraCaS textual inference problem set [4, 9]:

There was an Italian who became the world’s greatest tenor.

Such a simple case was analyzed in [7]. But even a slightly more complicated problem 018 below requires quite a non-trivial entailment

reasoning involving the relative clause.

- (1) Every European has the right to live in Europe.
- (2) Every European is a person.
- (3) Every person who has the right to live in Europe  
can travel freely within Europe.
- (4) Every European can travel freely within Europe.

As in all FraCaS problems, the goal is to determine the entailment of the last sentence (in our case, (4)) from the others. We must stress that FraCaS collects not only positive examples of expected entailments, but also negative examples where entailment does not hold – and also “yes and no” cases where entailment comes through only on some readings. Our goal is hence not only to derive entailments where expected, but also to explain why entailment does not hold in negative examples, as well as to reproduce several readings where present.

FraCaS has quite a few problems similar to the above, with copula relative clauses (problems 005, 006, 028) and quantifiers like ‘most’ (problem 074). Object relative clauses also appear (e.g., problems 133 and 344):

There is someone whom Helen saw answer the phone.  
ITEL maintains all the computers that GFI owns.

There are further complications, with quantified or coordinated relative clauses:

- (5) There was one auditor who signed all the reports.
- (6) There is a car that John and Bill own.
- (7) There is a representative that Smith wrote to every week.

We take FraCaS as a whetstone of a semantic theory, as a necessary application – but by no means sufficient. For example, NPs of the following form are quite common, but do not appear in FraCaS:

- (8) two students who skipped three classes
- (9) every student who skipped no classes
- (10) a student who didn’t skip all classes

One should be able to analyze them and derive entailments. A reviewer has pointed further interesting examples, such as

(11) land he had created and lived in

The present paper gives analysis of all such sentences and NPs, focusing on entailments.

## 2 Background

First a brief reminder of polynomial event semantics. It deals with events, notated  $e$ , such as ‘having become the world’s greatest tenor’ or ‘being Italian’ or ‘having the right to live in Europe’ (we denote the latter set of events as **RtlE**). It should be clear that we take events in a broad sense (as detailed in [10]): associated not only with actions but also states. Besides events, there are also individuals, notated  $i$ , such as **john**, and relations between events and individuals (written as **rel'**) such as

$$\text{subj}' = \{(e, i) \mid \text{ag}(e) = i\} \quad \text{obl}' = \{(e, i) \mid \text{th}(e) = i\}$$

where **ag** and **th** are thematic functions (for subjects and direct objects, resp.). Their names are mere the nod to the tradition in the event semantics literature (see §7).

If **rel'** is a relation of events to individuals, then

$$\text{rel}' / i \triangleq \{e \mid (e, i) \in \text{rel}'\}$$

is the set of events related to  $i$ . In particular,

$$\text{subj}' / \text{john} = \{e \mid (e, \text{john}) \in \text{subj}'\} = \{e \mid \text{ag}(e) = \text{john}\}$$

is the set of events whose subject is **john**. The semantics of a simple sentence such as “John has the right to live in Europe” is given compositionally as

$$(12) \quad \llbracket \text{John has the right to live in Europe} \rrbracket = \text{subj}' / \text{john} \cap \text{RtlE}$$

Subject, predicate, complements all denote event sets, and the whole sentence is their intersection. In particular, our sentence denotes –

or, is witnessed by – the events of having the right to live in Europe whose subject is John. The denotation is hence an event set – or the formula representing it, as (12), which one may think of as a query of the record of events in the world. A sentence is true in that world just in case the result of the query is a non-empty event set.

The denotation of the subject is also determined compositionally, by applying **subj'** to the denotation of NP, in our case, **john**.

If  $d_1$  and  $d_2$  are two denotations (queries), we say  $d_1$  entails  $d_2$  (in symbols:  $d_1 \implies d_2$ ) just in case for any record of events (for any world), whenever  $d_1$  delivers a non-empty set of witnessing events, so does  $d_2$ . If  $d_1$  and  $d_2$  entail each other, they are called equivalent.

## 2.1 Coordination and Quantification

We often deal not with individuals but with sets of individuals such as **Student** or **European**, which are the denotations of common nouns. Determiners pick which individuals from this set to consider. Correspondingly, they call for generalization: the introduction of (internal) choice  $\sqcup$  (for narrow-scope existentials and indefinites), external choice  $\oplus$  (wide-scope ones) and grouping  $\otimes$ . Thus **john**  $\sqcup$  **bill** is a choice between John and Bill, whereas **john**  $\otimes$  **bill** is a group of John and Bill: both have to be involved, not necessarily in the same action however. Likewise, event sets are generalized to *polyconcepts*, such as  $d_1 \otimes d_2$  for two disjoint event sets  $d_1$  and  $d_2$ , which specifies that an event from  $d_1$  *and* an event from  $d_2$  must have transpired. Grouping is quite different from intersection  $\cap$  (generalized to  $\sqcap$  for polyconcepts), which describes common events. In particular, if  $d$  is a singleton event set, then  $d \otimes d = \perp$  (the empty polyconcept), but  $d \sqcap d = d$ .

We define for convenience

$$\mathcal{E}c = \sqcup_{j \in c} j \qquad \mathcal{I}c = \oplus_{j \in c} j \qquad \mathcal{A}c = \otimes_{j \in c} j$$

The meaning of “All Europeans/Every European” is then  $\mathcal{A}$  **European**; on the other hand,  $\llbracket$ A **European** $\rrbracket$  (narrow scope) is  $\mathcal{E}$  **European**. Therefore, (1), repeated below

- (1) Every European has the right to live in Europe.

has as its denotation

$$\begin{aligned}
& (\text{subj}' / \mathcal{A}\text{European}) \sqcap \text{RtIE} \\
& = \mathcal{A}(\text{subj}' / \text{European}) \sqcap \text{RtIE} \\
(13) \quad & = \bigotimes_{i \in \text{European}} (\text{subj}' / i \cap \text{RtIE})
\end{aligned}$$

where  $\sqcap$  is the generalization of set intersection  $\cap$  to polyconcepts;  $\text{subj}' /$  is likewise generalized to apply to sets of individuals and poly-individuals – as homomorphism.

The distribution laws detailed in [7] lead to (13), which asserts there is a group of non-empty events of having right to live in Europe, and each European is a subject of some event in that group.

The notion of entailment extends to polyconcepts: The polyconcept  $d_1$  entails  $d_2$  just in case  $d_1 \neq \perp \implies d_2 \neq \perp$ . For example, [7] described several equational laws of polyconcepts, among whose is

$$x \otimes \perp = \perp \otimes x = \perp$$

from which it logically follows that

$$(14) \quad x \otimes y \neq \perp \implies x \neq \perp$$

That is, (13) entails  $\text{subj}' / i \cap \text{RtIE}$  for any individual  $i \in \text{European}$ . In other words, if John is European, (1) entails that “John has the right to live in Europe”.

## 2.2 Negation

The central idea of polynomial event semantics is that the sentence denotation is a query (formula), which when applied to the record of events in a world, selects the events that support, or witness, the sentence. If that set of events turns out empty, the sentence is not supported (in that world). Negation hence presents a problem: what is the witness for the absence of support?

Our resolution [6] is to consider counter-examples. The denotation of a sentence with negation or negative quantification is also a query, but what it selects is interpreted as counter-examples. If the set of counter-examples is empty, the corresponding sentence is not

contradicted. To distinguish counter-example-selecting queries and polyconcepts, they are marked with the  $\neg$  sign.

For example, the denotation of (15) below is (16), which can be re-written to (17) according to the laws of [6].

(15) John didn't skip the PE class.

(16)  $\text{subj}' / \text{john} \sqcap \neg \text{Skip} \sqcap \text{ob1}' / \text{peClass}$

(17)  $= \neg (\text{subj}' / \text{john} \sqcap \text{Skip} \sqcap \text{ob1}' / \text{peClass})$

Likewise, the denotation for (18) is (19):

(18) John didn't skip every class.

(19)  $\neg \bigotimes_{i \in \text{Class}} (\text{subj}' / \text{john} \sqcap \text{Skip} \sqcap \text{ob1}' / i)$

Indeed, the counter-example for (18) would be a group of John skipping class events, for each class.

Negation calls for further generalization of entailment. If  $d_1$  and  $d_2$  are polyconcepts marked as negative,  $d_1 \implies d_2$  just in case  $d_2 \neq \perp \implies d_1 \neq \perp$ : whenever  $d_2$  is contradicted, then so is  $d_1$ . We thus obtain that (15) entails (18).

Suppose that **Attend** and **Skip** are disjoint event sets. Then if “John attended the PE class” is supported, “John didn't skip the PE class” cannot be contradicted. For the sake of such entailments, we introduce (see [6, 7])

$$(20) \text{ justified}(d) = \begin{cases} d = \perp & \text{if } d \text{ is marked as negative} \\ d \neq \perp & \text{if } d \text{ is not negatively marked} \end{cases}$$

which lets us define entailment most generally:  $d_1$  entails  $d_2$  just in case  $\text{justified}(d_1)$  logically entails  $\text{justified}(d_2)$ . For example, we may now derive that “John attended the PE class” entails (18).

### 3 Subject Relative Clauses

The problem is hence determining the meaning of “who has the right to live in Europe.” If **RtlE** is the set of events of having the right to live in Europe, then who has that right is the subject of these events.

Thus the denotation of our subject relative clause, to be notated as  $\overline{\text{subj}}' / \text{RtlE}$ , is the set of individuals

$$(21) \quad \overline{\text{subj}}' / \text{RtlE} \triangleq \{\text{ag}(e) \mid e \in \text{RtlE}\}$$

Then (3), recalled below,

- (3) Every person who has the right to live in Europe  
can travel freely within Europe.

has as its denotation

$$(22) \quad \text{subj}' / \mathcal{A} \left( \text{Person} \cap (\overline{\text{subj}}' / \text{RtlE}) \right) \sqcap \text{CtfE}$$

where  $\text{CtfE}$  is the set of events of having the possibility to freely travel within Europe. This analysis is more or less what was described in [7], but recast now in simpler terms. It takes us quite far: many more FraCaS problems can be analyzed similarly.

However, relative clauses with quantifiers, coordination or negation present a problem. Again we need to generalize. Remembering the definition of the  $\text{subj}'$  relation, we may re-write (21) as

$$\begin{aligned} \overline{\text{subj}}' / \text{RtlE} &\triangleq \{\text{ag}(e) \mid e \in \text{RtlE}\} \\ &= \{i \mid e \in \text{RtlE}, (i, e) \in \text{subj}'\} \\ &= \{i \mid \text{subj}' / i \cap \text{RtlE} \neq \emptyset\} \end{aligned}$$

One may notice that  $\text{subj}' / i \cap \text{RtlE}$  is exactly the meaning of “ $i$  has the right to live in Europe”. That is, “who has the right to live in Europe” is the set of those  $i$  in the domain of the  $\text{subj}'$  who make the sentence true (in the world of the discourse). The denotation of a subject relative clause “who  $C$ ” may hence be defined as

$$(23) \quad \llbracket \text{who } C \rrbracket \triangleq \overline{\text{subj}}' / \llbracket C \rrbracket = \{i \mid \text{subj}' / i \cap \llbracket C \rrbracket \neq \emptyset\}$$

This is already helpful, to solve FraCaS 018, which is, recall

- (1) Every European has the right to live in Europe.
- (2) Every European is a person.
- (3) Every person who has the right to live in Europe  
can travel freely within Europe.
- (4) Every European can travel freely within Europe.

For (1) we have earlier obtained the denotation (13), which says that  $\text{subj}'/i \cap \text{RtlE}$  is non-empty for all  $i \in \text{European}$ . Then (23) immediately gives  $\text{European} \subset \overline{\text{subj}}'/\text{RtlE}$ ; in words: the set of who has the right to live in Europe includes all Europeans. Likewise, (2) gives  $\text{European} \subset \text{Person}$ , leading to  $\text{European} \subset (\text{Person} \cap \overline{\text{subj}}'/\text{RtlE})$ . Then, by monotonicity of  $\mathcal{A}$ , (22) entails  $\text{subj}'/(\mathcal{A}\text{European}) \sqcap \text{CtfE}$ , which is the denotation of (4). The entailment of (4) from the other sentences of the problem indeed holds.

Definition (23), unlike (21), now easily generalizes to the case when the denotation of the rest of the clause  $d$  is not an event set but a polyconcept with choice or grouping:

$$(24) \quad \llbracket \text{who } C \rrbracket \triangleq \overline{\text{subj}}'/\llbracket C \rrbracket = \{i \mid \text{subj}'/i \sqcap \llbracket C \rrbracket \neq \perp\}$$

The generalization lets us analyze quantified and coordinated relative clauses such as (5)-(7). For example, for (5), repeated below,

(5) There was one auditor who signed all the reports.

we obtain the denotation ( $\mathcal{E}\text{Be}$  is an existence event, see [7])

$$\text{subj}'/\mathcal{E} \left( \text{Auditor} \cap \overline{\text{subj}}'/(\text{Sign} \sqcap \text{ob1}'/\mathcal{A}\text{Report}) \right) \sqcap \mathcal{E} \text{Be}$$

where, to remind,  $\overline{\text{subj}}'/(\text{Sign} \sqcap \text{ob1}'/\mathcal{A}\text{Report})$  is a set of those individuals  $i$  such that

$$\llbracket i \text{ signed all the reports} \rrbracket = \text{subj}'/i \sqcap \text{Sign} \sqcap \text{ob1}'/\mathcal{A}\text{Report}$$

is justified. By the very construction, the scope of the universal does not extend past its clause. We return to this example in §5 and show a more intuitive, and useful for entailment, analysis.

## 4 Other Relative Clauses

The approach introduced in §3 easily extends to object, locative, etc. relative clauses. A good example to illustrate is as follows, also containing an interesting case of coordination:<sup>1</sup>

The land over which he sped was  
the land he had created and lived in: his valley.

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<sup>1</sup> from Clifford Irving, *The Valley* (1961) – sentence 170 of `susanne_N02` included in [2].



We concentrate on one constituent:

(25) land he had created and lived in

Section 2 already introduced the relation  $\mathbf{ob1}'$  between an action and its direct object; analogous to it is the relation  $\mathbf{inloc}'$  between an action and an individual denoting location. Let  $\mathbf{Land}$  be the set of such location-individuals. Similarly to (24) we may then define

$$(26) \quad \llbracket \text{which } C \rrbracket \triangleq \overline{\mathbf{ob1}'} / \llbracket C \rrbracket \triangleq \{i \mid \mathbf{ob1}' / i \sqcap \llbracket C \rrbracket \neq \perp\}$$

$$(27) \quad \llbracket \text{in which } C \rrbracket \triangleq \overline{\mathbf{inloc}'} / \llbracket C \rrbracket \triangleq \{i \mid \mathbf{inloc}' / i \sqcap \llbracket C \rrbracket \neq \perp\}$$

which gives us

$$\begin{aligned} \llbracket \text{land he had created} \rrbracket &= \mathbf{Land} \cap \overline{\mathbf{ob1}'} / (\mathbf{subj}' / \mathbf{He} \sqcap \mathbf{Created}) \\ (28) \quad &= \mathbf{Land} \cap \{i \mid \mathbf{subj}' / \mathbf{He} \sqcap \mathbf{Created} \sqcap \mathbf{ob1}' / i \neq \perp\} \end{aligned}$$

$$\begin{aligned} \llbracket \text{land he had lived in} \rrbracket &= \mathbf{Land} \cap \overline{\mathbf{inloc}'} / (\mathbf{subj}' / \mathbf{He} \sqcap \mathbf{Lived}) \\ (29) \quad &= \mathbf{Land} \cap \{i \mid \mathbf{subj}' / \mathbf{He} \sqcap \mathbf{Lived} \sqcap \mathbf{inloc}' / i \neq \perp\} \end{aligned}$$

where  $\mathbf{He}$  is a particular individual to which the pronoun “he” is resolved. The repetitiveness and boilerplate are apparent: generalization is in order.

We have been assigning the denotation to a sentence in a surface form – at least, how it appeared so far. In reality, we take as input a parse tree, with Penn-treebank-like annotations (see [2]). For example, “land he had created” and “land he had lived in” are represented as:

$$(30) \quad \text{land}_N \left[ \text{IP-REL} \left[ \text{NP-SBJ } \text{he}_{\text{PRO}} \right] \text{had}_{\text{HVD}} \left[ \text{NP-OBJ } \mathbf{T} \right] \text{created}_{\text{VFN}} \right]$$

$$(31) \quad \text{land}_N \left[ \text{IP-REL} \left[ \text{NP-SBJ } \text{he}_{\text{PRO}} \right] \text{had}_{\text{HVD}} \text{lived}_{\text{VFN}} \left[ \text{PP-LOC } \text{in}_{\text{P-ROLE}} \left[ \text{NP } \mathbf{T} \right] \right] \right]$$

where  $\mathbf{T}$  is the trace. Denotations (28) and (29) then clearly correspond to the annotated trees (30) and (31), resp. In fact, they are built as follows:

$$\begin{aligned} \llbracket \text{land}_N \rrbracket &= \mathbf{Land} & \llbracket \text{created}_{\text{VFN}} \rrbracket &= \mathbf{Created} & \llbracket \text{lived}_{\text{VFN}} \rrbracket &= \mathbf{Lived} \\ \llbracket \left[ \text{NP-SBJ } \text{he}_{\text{PRO}} \right] \rrbracket &= \mathbf{subj}' / \llbracket \text{he}_{\text{PRO}} \rrbracket & &= \mathbf{subj}' / \mathbf{He} \\ \llbracket \left[ \text{NP-OBJ } i \right] \rrbracket &= \mathbf{ob1}' / i & \llbracket \left[ \text{PP-LOC } \text{in}_{\text{P-ROLE}} \left[ \text{NP } i \right] \right] \rrbracket &= \mathbf{inloc}' / i \\ (32) \quad \llbracket \left[ \text{IP-REL } C \right] \rrbracket &= \{i \mid \llbracket C/i \rrbracket \neq \perp\} \end{aligned}$$

where  $C/i$  means replacing the trace with  $i$ . (Aspect/tense is out of scope, and the auxiliary  $\text{had}_{\text{HVD}}$  is ignored.) Connections of any constituents are uniformly represented as the intersection ( $\cap$  or  $\sqcap$ ) of their denotations. Therefore, for the original clause (25) we have:<sup>2</sup>

$$(33) \llbracket (25) \rrbracket = \text{Land} \cap \{i \mid \text{subj}'/ \text{He} \sqcap ((\text{Created} \sqcap \text{ob1}'/ i) \otimes (\text{Lived} \sqcap \text{inloc}'/ i)) \neq \perp\}$$

which lets us do entailments. For example, from (28) and (14), we obtain that

$$\llbracket (25) \rrbracket \subset \llbracket \text{the land he created} \rrbracket$$

There remains a puzzle, however: (32) seems postulated, with the operation  $C/i$  of trace substitution coming out of the blue. It is also hardly compositional. We now describe how (32) comes about, and how to derive denotations (28), (29) and (33) and others like them rigorously and compositionally. We will also see what is the denotation of trace after all.

#### 4.1 Relative Algebra

Recall, to give denotations in the polynomial event semantics we use the algebra of polyconcepts (denoted by the metavariable  $d$ ) with the operations  $\perp$ ,  $\oplus$ ,  $\otimes$ ,  $\sqcap$  and  $\sqcup$ . Strictly speaking, there are two algebras: their operations are the same, but generators differ: individuals vs. event sets. The two algebras are homomorphic: the operations  $\text{subj}'/ \cdot$ ,  $\text{ob1}'/ \cdot$ , etc. are the homomorphisms.

We now introduce yet another algebra – called relative algebra – whose carrier are relations between individuals and polyconcepts: sets of pairs  $(i, d)$  where  $i$  is an individual ranging over  $\mathbb{I}$ , the set of all individuals. The operations of the relative algebra are the lifted operations on polyconcepts:  $\perp$  of relative algebra is  $\{(i, \perp) \mid i \in \mathbb{I}\}$ , and

$$\{(i, d_1) \mid i \in \mathbb{I}\} \sqcap \{(i, d_2) \mid i \in \mathbb{I}\} = \{(i, d_1 \sqcap d_2) \mid i \in \mathbb{I}\}$$

and similarly for the other operations. Relative algebra is clearly homomorphic to a polyconcept algebra: for each polyconcept  $d$  there

<sup>2</sup> the annotated form is [https://oncoj.orinst.ox.ac.uk/cgi-bin/tspc.sh?tree=170\\_susanne\\_N02@21&mode=clip](https://oncoj.orinst.ox.ac.uk/cgi-bin/tspc.sh?tree=170_susanne_N02@21&mode=clip)

is a corresponding (we say, ‘lifted’) element of the relative algebra:

$$d \rightarrow \{(i, d) \mid i \in \mathbb{I}\}$$

with the algebra operations commuting with lifting, as we already observed. Relative algebra also has elements that are not lifted poly-concepts. Among them is so-called  $t^r$ :

$$t^r \triangleq \{(i, i) \mid i \in \mathbb{I}\}$$

Let  $\llbracket C \rrbracket_{rel}$  be the denotation of a clause/constituent  $C$  in terms of relative algebra. It is built compositionally, mostly from the lifted denotations of its sub-constituents; the exception is the denotation of trace, which is  $t^r$ . The denotation of a relative clause is then

$$(34) \quad \llbracket [\text{IP-REL } C] \rrbracket \triangleq \{i \mid (i, d) \in \llbracket C \rrbracket_{rel}, d \neq \perp\}$$

This is the compositional analogue of (32): with it and the denotation of trace as  $t^r$ , we may build denotations of arbitrary relative clauses. In particular, the denotation of (25) works out to be exactly (33).

## 5 Relative Clauses, Database Joins, and Trace as a Wide-scope Indefinite

We now show two other, related, points of view on relative clauses. One treats relative clauses as database joins. The other regards trace as a wide-scope indefinite, and separates out the relative clause into an independent sentence, to which the original sentence anaphorically refers.

Relative clauses are NP modifiers: for example, in “land (that) he created”, the relative clause modifies the common noun “land”. As described earlier, polynomial event semantics derives the compositional denotation

$$(35) \llbracket \text{land}_N [\text{IP-REL he created T}] \rrbracket = \llbracket \text{land}_N \rrbracket \cap \llbracket [\text{IP-REL he created T}] \rrbracket$$

(The denotation of the relative clause is also derived compositionally per (34).) Since the trace is related to “land”, one may wonder about a way to reflect that relation in the denotation. Although this breaks compositionality to some (small) extent, the insight seems worth it.

Among operations of the (relative) polyconcept algebra,  $\oplus$  stands out: it distributes across/commutes with any other operation. For example, letting  $d_{hc}$  be  $\text{subj}' / \text{He} \sqcap \text{Created}$ ,

$$(36) \quad \llbracket \text{He created a}_W \text{ land} \rrbracket = d_{hc} \sqcap \text{ob1}' / \mathcal{I} \text{Land} \\ \equiv d_{hc} \sqcap \text{ob1}' / \left( \bigoplus_{j \in \text{Land}} j \right) = \bigoplus_{j \in \text{Land}} (d_{hc} \sqcap \text{ob1}' / j)$$

where  $\text{a}_W$  is a wide-scope indefinite. Let

$$\delta_{ij} = \begin{cases} i & \text{if } i = j \\ \perp & \text{otherwise} \end{cases} = i \sqcap j$$

which is a bona fide polyconcept. Let us introduce a ‘lifted’  $\mathcal{I}$ :

$$\mathcal{I}^r S \triangleq \{(i, \bigoplus_{j \in S} \delta_{ij}) \mid i \in \mathbb{I}\}$$

and note that

$$(37) \quad \{(i, d_{hc}) \mid i \in \mathbb{I}\} \sqcap \text{ob1}' / \mathcal{I}^r \text{Land} \\ = \{(i, \bigoplus_{j \in \text{Land}} (d_{hc} \sqcap \text{ob1}' / \delta_{ij})) \mid i \in \mathbb{I}\}$$

looks like a relative algebra denotation of some clause, which we call  $C_{hcal}$  for the time being. Treating it as if it were a relative clause

$$\begin{aligned} \llbracket [\text{IP-REL } C_{hcal}] \rrbracket &= \{i \mid (i, d) \in \llbracket C_{hcal} \rrbracket_{rel}, d \neq \perp\} \\ &= \{i \mid \bigoplus_{j \in \text{Land}} (d_{hc} \sqcap \text{ob1}' / \delta_{ij}) \neq \perp\} \\ &= \{i \mid i \in \text{Land}, d_{hc} \sqcap \text{ob1}' / i \neq \perp\} \\ &= \text{Land} \cap \llbracket [\text{IP-REL he created T}] \rrbracket \end{aligned}$$

gives an interesting result: on one hand, the denotation of “land he created” may be computed compositionally per (35), from  $\llbracket \text{land}_N \rrbracket$  and the denotation of the proper relative clause (with trace interpreted as  $t^r$ ). On the other hand, the entire  $\llbracket \text{land}_N [\text{IP-REL he created T}] \rrbracket$  can be computed in one scoop, as the denotation of a “relative clause”  $C_{hcal}$ , as we have just shown. What is  $C_{hcal}$  then? Comparing (37) with (36) we notice they are almost the same: only the former uses relative algebra, and instead of  $\text{a}_W$ , denoted by  $\mathcal{I}$ , we have something else, denoted by  $\mathcal{I}^r$ . One may call it  $\text{a}_W^r$ : a wide scope

indefinite to which one may refer to. We thus obtained that “land he created” – the relative clause together with the modified noun – is related to an independent sentence “he created  $a_W^r$  land”. In fact, it is a set of referents created by the indefinite  $a_W^r$  of that sentence.

Hence, speaking in database terms, relative clause is a join. Using a FraCaS example, the denotation of (3) may be regarded as a database join, of “A person has the right to live in Europe.” with “can travel freely within Europe” on subject. Such database join may be illustrated by a (bit contrived) paraphrase: “Some people have the right to live in Europe. Every one of them can travel freely within Europe.” That is, the relative clause is moved out into a separate sentence, with the trace filled with a (wide-scope) indefinite. The original sentence anaphorically refers to that indefinite. One can generalize: “It builds up muscles people thought didn’t exist.” to paraphrase as “People thought some muscles didn’t exist. It builds up them.”

This replacement of trace with the indefinite leads to a variety of analyses. Returning to (5), repeated below,

(5)      There was one auditor who signed all the reports.

we obtain the denotation

$$\text{subj}' / \mathcal{E} \llbracket \text{auditor who signed all the reports} \rrbracket \sqcap \mathcal{E} \text{ Be}$$

where  $\mathcal{E} \text{ Be}$  is an existence event (see [7]). This denotation is equivalent to  $\llbracket \text{auditor who signed all the reports} \rrbracket$ , which is equivalent to  $\llbracket a_W^r \text{ auditor signed all the reports} \rrbracket$ . In other words, (5) is equivalent to, or mutually entails, “One particular auditor/the same auditor signed all the reports” – which is what FraCaS problem 196 is all about.

Similarly, we obtain that (6) is equivalent to “John and Bill own the same car”. For problem 308, we obtain (7) is equivalent to “Smith wrote to a representative every week.” on the wide-scope reading of the indefinite – with no entailment for the narrow-scope reading.

## 6 Negation in Relative Clauses

Negation calls for one more generalization of (24) and related denotations:

$$\llbracket \text{who } C \rrbracket \triangleq \overline{\text{subj}}' / \llbracket C \rrbracket = \{i \mid \text{justified}(\text{subj}' / i \sqcap \llbracket C \rrbracket)\}$$

We calculate, for example

$$\begin{aligned} & \llbracket \text{student who didn't skip all classes} \rrbracket \\ &= \text{Student} \cap \overline{\text{subj}}' / \llbracket \text{didn't skip all classes} \rrbracket \\ &= \text{Student} \cap \{i \mid \text{justified}(\text{subj}' / i \sqcap (\neg \text{Skip} \sqcap \text{ob1}' / \mathcal{A}\text{Class}))\} \\ &= \text{Student} \cap \{i \mid \text{justified}(\neg(\text{subj}' / i \sqcap \text{Skip} \sqcap \text{ob1}' / \mathcal{A}\text{Class}))\} \\ &= \text{Student} \cap \{i \mid \neg(\text{subj}' / i \sqcap \text{Skip} \sqcap \text{ob1}' / \mathcal{A}\text{Class}) = \perp\} \\ &= \text{Student} \cap \{i \mid \text{subj}' / i \sqcap \text{Skip} \sqcap \text{ob1}' / \mathcal{A}\text{Class} = \perp\} \end{aligned}$$

by unrolling definitions and applying the distributive laws of negation touched upon in §2.2. Comparing with

$$\begin{aligned} & \llbracket \text{students who skipped all classes} \rrbracket \\ &= \text{Student} \cap \{i \mid \text{justified}(\text{subj}' / i \sqcap \text{Skip} \sqcap \text{ob1}' / \mathcal{A}\text{Class})\} \\ &= \text{Student} \cap \{i \mid \text{subj}' / i \sqcap \text{Skip} \sqcap \text{ob1}' / \mathcal{A}\text{Class} \neq \perp\} \end{aligned}$$

we easily see that the two sets are complementary. Likewise, “student who skipped no classes” is the complement of the set of students who skipped a class.

## 7 Related Work, Discussion and Conclusions

Semantic and syntactic analyses are often tightly coupled: e.g., lexical entries are assigned syntactic categories or features, as well as semantic interpretations (often as lambda-terms). Examples include various categorial-grammar-based analyses (see [8] for the latest example), minimalist grammars ([12] for the latest), etc. As a result, semantic analysis is coupled to a particular parsing technique. In contrast, we, like Bulter’s Treebank semantics [1], start with an already parsed sentence: to be precise, Penn-treebank-like annotated tree (see [2] for details on annotations). The annotations can be assigned

manually or by any appropriate parser (e.g., Stanford CoreNLP [1]). Starting from an annotated sentence is also common in dependency-tree-based approaches, see [11]. (Our approach can also be adapted to dependency trees).

Closest to our work is the dependency-tree semantics of Tian et al. [11], who also represent the meaning of a sentence as an abstract query. The paper [11] briefly mentions relative clauses, analyzed along the lines of (21). Our and dependency-tree semantics diverge when it comes to quantification and coordination: we depart relational algebra for polyconcepts, expressing grouping and choice.

Analyses of relative clauses in event semantics are rare. One of the few is the relatively recent [12, §4.2.1 and §6], which uses Minimalist Grammar coupled with a continuation-based approach (in the spirit of [3]).

As a nod to the tradition, §2 mentioned thematic functions **ag** and **th** when defining the **subj'** and **ob1'** relations. These relations are meant to be grammatical subject and object relations, with **ag** specifying the grammatical subject of an action carried by a verb, rather than the semantic agent. After all, event semantics is widely praised for avoiding meaning postulates as far as possible and deriving entailments from the structure alone. Likewise, the focus of FraCaS is textual entailment without relying on world knowledge. We, too, concentrate on the structure: Just as verbs have arguments, events – records in a world database – have attributes. The functions **ag** and **th**, etc. merely refer to these attributes. As a consequence, we treat active and passive VP as completely separate, and do not consider entailments between active and passive forms of the same verb. In the future, we may introduce a postulate that, say, for any event  $e \in \mathbf{See}$  there exists an event  $e' \in \mathbf{BeSeen}$  such that  $\mathbf{ag}(e') = \mathbf{th}(e)$  and  $\mathbf{intstr}(e') = \mathbf{ag}(e)$ . One may also deal with passive constructions syntactically: convert passive construction to active at parsing time (see Prithiviraj Damodaran’s Styleformer based on Stanford CoreNLP.)

In conclusion, we demonstrated handling of relative clauses in polynomial event semantics, from simple to coordinated and quantified. The approach handles the subject, object, locative, etc. relative clauses. Extension to tense/time is straightforward. Future work is

the mechanical implementation of the approach to derive the entailments automatically.

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